Math 1251: Worksheet #3 **Winter 2018**

Name: SOLUTIONS	A#:	Section:

1. Find the volume of the solid obtained by revolving the segment of the curve $y = x + \frac{1}{x}$ between $x = 1$ and $x = 2$ about the x-axis.

$$\text{Volume} = \int_1^2 \pi \left(x + \frac{1}{x}\right)^2 dx$$
$$= \pi \int_1^2 \left(x^2 + 2 + \frac{1}{x^2}\right) dx$$
$$= \pi \left(\frac{1}{3}x^3 + 2x - \frac{1}{x}\right)\Big]_1^2$$
$$= \pi \left[\left(\frac{8}{3} + 4 - \frac{1}{2}\right) - \left(\frac{1}{3} + 2 - 1\right)\right]$$
$$= \boxed{\frac{29\pi}{6}}$$

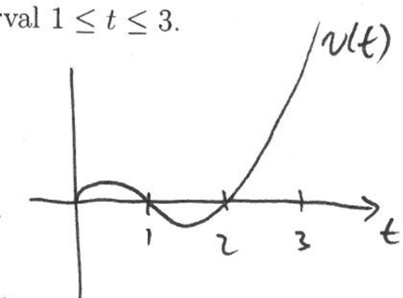
2. A particle moves in a straight line so that its velocity after t seconds is $v(t) = t^3 - 3t^2 + 2t$ metres per second.

(a) Find the net displacement of the particle over the time interval $1 \le t \le 3$.

$$\text{Displacement} = \int_1^3 (t^3 - 3t^2 + 2t)\,dt$$
$$= \left(\frac{1}{4}t^4 - t^3 + t^2\right)\Big|_1^3$$
$$= \left(\frac{81}{4} - 27 + 9\right) - \left(\frac{1}{4} - 1 + 1\right)$$
$$= \frac{9}{4} - \frac{1}{4} = \boxed{2}$$

(b) Find the total distance travelled by the particle over the time interval $1 \le t \le 3$.

Solve $0 = t^3 - 3t^2 + 2t$
$= t(t^2 - 3t + 2)$
$= t(t-1)(t-2) \Rightarrow t = 0, 1, 2.$



$$\text{Distance} = -\int_1^2 (t^3 - 3t^2 + 2t)\,dt + \int_2^3 (t^3 - 3t^2 + 2t)\,dt$$
$$= -\left(\frac{1}{4}t^4 - t^3 + t^2\right)\Big|_1^2 + \left(\frac{1}{4}t^4 - t^3 + t^2\right)\Big|_2^3$$
$$= -\left[0 - \frac{1}{4}\right] + \left[\frac{9}{4} - 0\right]$$
$$= \frac{10}{4}$$
$$= \boxed{\frac{5}{2}}$$

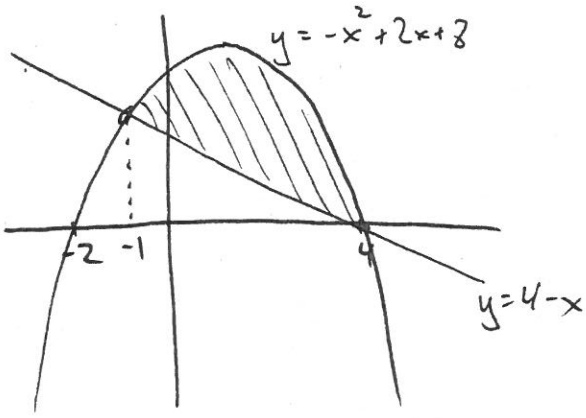
3. Sketch the curves $y = 4 - x$ and $y = -x^2 + 2x + 8$ and find the area bounded between them. (Show all your work.)

Roots of quadratic: $0 = -x^2 + 2x + 8 = -(x-4)(x+2)$ $\Rightarrow$ Roots $x = -2$, $x = 4$

Intersection pts: $4 - x = -x^2 + 2x + 8$

$\Leftrightarrow \quad 0 = x^2 - 3x + 4$

$\Leftrightarrow \quad 0 = (x-4)(x+1) \Leftrightarrow x = 4$ or $x = -1$



$$\text{Area} = \int_{-1}^{4} \left(-x^2 + 2x + 8 - (4 - x) \right) dx$$

$$= \int_{-1}^{4} \left(-x^2 + 3x + 4 \right) dx$$

$$= \left(-\frac{1}{3}x^3 + \frac{3}{2}x^2 + 4x \right) \Big|_{-1}^{4}$$

$$= \left(-\frac{64}{3} + 24 + 16 \right) - \left(\frac{1}{3} + \frac{3}{2} - 4 \right)$$

$$= -\frac{65}{3} - \frac{3}{2} + 44$$

$$= \frac{-130 - 9 + 264}{6} = \boxed{\frac{125}{6}}$$

4. Find all k such that the average value of $f(x) = ke^{x/2} - 1$ over $[0, 2]$ is equal to 2.

The average value is $\frac{1}{2-0} \int_0^2 \left(ke^{x/2} - 1 \right) dx$

$$= \frac{1}{2} \left(2ke^{x/2} - x \right) \Big|_0^2$$

$$= \frac{1}{2} \left(2ke - 2 - (2k - 0) \right)$$

$$= ke - 1 - k.$$

So we need $ke - 1 - k = 2 \Rightarrow k(e-1) = 3$

$$\Rightarrow \boxed{k = \frac{3}{e-1}}$$