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Math 1251: Worksheet #4 **Winter 2018**

Name: SOLUTIONS	A#:	Section:

1. Let $f(x,y) = x^3 e^{z+xy^2} + (x-z)^3$.

(a) Compute $\dfrac{\partial f}{\partial x}$

$$3x^2 e^{z+xy^2} + x^3 e^{z+xy^2} \cdot y^2 + 3(x-z)^2$$

(3)

$$= x^2 e^{z+xy^2}(3+xy^2) + 3(x-z)^2$$

(b) Compute $\dfrac{\partial f}{\partial y}$

$$x^3 e^{z+xy^2} \cdot 2xy$$

(3)

$$= 2x^4 y e^{z+xy^2}$$

(c) Compute $\dfrac{\partial f}{\partial z}$

$$x^3 e^{z+xy^2} + 3(x-z)^2(-1)$$

(3)

(d) Verify that $\dfrac{\partial^2 f}{\partial y\,\partial x} = \dfrac{\partial^2 f}{\partial x\,\partial y}$.

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\frac{\partial f}{\partial x} = \frac{\partial}{\partial y}\left[x^2 e^{z+xy^2}(3+xy^2) + 3(x-z)^2\right]$$

$$= x^2 e^{z+xy^2} \cdot 2xy \cdot (3+xy^2) + x^2 e^{z+xy^2}(0+2xy)$$

(5)

$$= 2x^3 y e^{z+xy^2}(4+xy^2)$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\frac{\partial f}{\partial y} = \frac{\partial}{\partial x}\left[2x^4 y e^{z+xy^2}\right]$$

$$= 8x^3 y e^{z+xy^2} + 2x^4 y e^{z+xy^2} \cdot y^2$$

$$= 2x^3 y e^{z+xy^2}(4+xy^2)$$

v4.1

2. Let $f(x,y) = x^2(1-xy)$. On separate pairs of axes, sketch the cross-sections $z = f(x,1)$ and $z = f(1,y)$.

$f(x,1) = x^2(1-x)$

$f(1,y) = 1-y$

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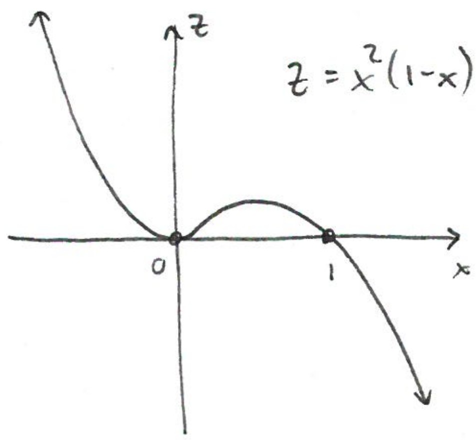
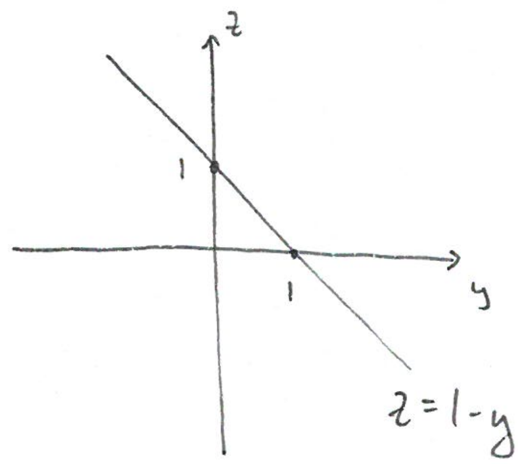




3. Let $f(x,y) = \dfrac{y-1}{x^2+1}$. Draw the level curves $f(x,y) = c$ for $c = 0, 1, 2$.

$f(x,y) = c$

$\Leftrightarrow \quad \dfrac{y-1}{x^2+1} = c$

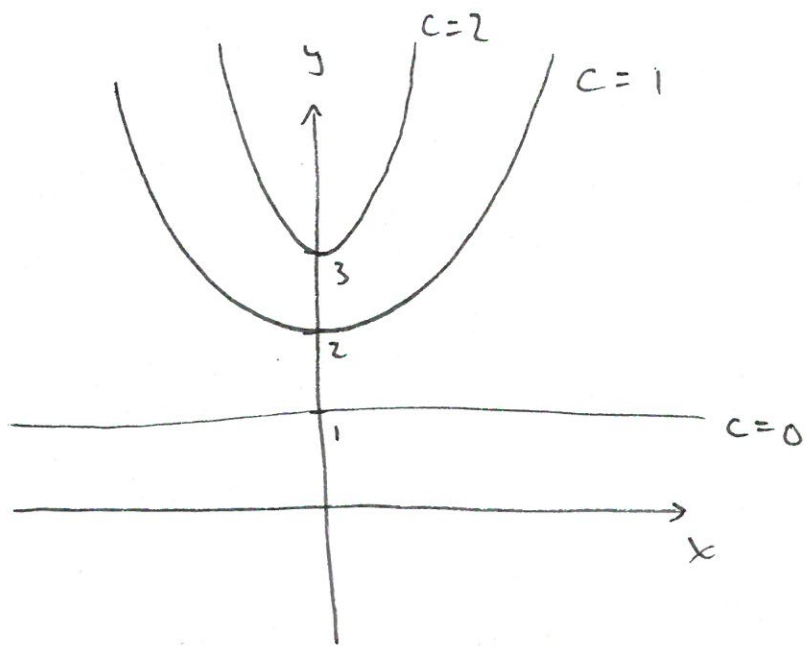
4 $\Leftrightarrow \quad y - 1 = c(x^2+1)$

$\Leftrightarrow \quad y = cx^2 + (c+1)$

$c=0: \quad y = 1$

$c=1: \quad y = x^2+2$

$c=2: \quad y = 2x^2+3$



4. Let $g(x,y) = (1 - x^3\sqrt{y})^4$. Use partial derivatives to estimate the value of $g(1+h, 4)$ for small h.

$$\frac{\partial g}{\partial x} = 4(1-x^3\sqrt{y})^3(-3x^2\sqrt{y}) \Rightarrow \frac{\partial g}{\partial x}(1,4) = 4(1-1\cdot 2)^3(-3\cdot 1^2\cdot 2)$$

$$= 24$$

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Then $g(1+h, 4) \approx g(1,4) + 24h$

$$= \boxed{1 + 24h}$$