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Math 1251: Worksheet #6 **Winter 2018**

Name: SOLUTIONS	A#:	Section:

1. Compute the following integrals:

(a) $\int \frac{dx}{x \ln x}$

$u = \ln x$
$du = \frac{1}{x}\,dx$

$= \int \frac{du}{u}$

4 $= \ln|u| + C$

$= \boxed{\ln|\ln x| + C}$

(b) $\int \frac{x^2}{(1-2x^3)^4}\,dx$

$u = 1 - 2x^3, \quad du = -6x^2\,dx$

4 $= -\frac{1}{6}\int \frac{du}{u^4}$

$= -\frac{1}{6}\left(-\frac{1}{3}u^{-3}\right) + C$

$= \boxed{\frac{1}{18} \cdot \frac{1}{(1-2x^3)^3} + C}$

(c) $\int \frac{dt}{\sqrt{t}e^{\sqrt{t}}}$

$u = \sqrt{t}, \quad du = \frac{1}{2\sqrt{t}}\,dt$

$\Rightarrow \frac{dt}{\sqrt{t}} = 2\,du$

4 $= \int \frac{2\,du}{e^u}$

$= 2\int e^{-u}\,du$

$= -2e^{-u} + C$

$= \boxed{-2e^{-\sqrt{t}} + C}$

(d) $\int_0^1 x\sqrt{1+3x}\,dx$

$u = 1+3x \Rightarrow x = \frac{u-1}{3}$
$du = 3\,dx$

$= \int_1^4 \frac{u-1}{3}\sqrt{u}\cdot\frac{du}{3}$

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$= \frac{1}{9}\int_1^4 (u-1)\sqrt{u}\,du$

$= \frac{1}{9}\int_1^4 \left(u^{3/2} - u^{1/2}\right) du$

$= \frac{1}{9}\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right)\Big|_1^4 = \frac{2}{9}\left(\left(\frac{32}{5}-\frac{8}{3}\right)-\left(\frac{1}{5}-\frac{1}{3}\right)\right) = \frac{2}{9}\left(\frac{31}{5}-\frac{7}{3}\right) = \frac{2}{9}\cdot\frac{58}{15}$

$= \boxed{\frac{116}{135}}$

2. Let R be the region of the (x, y)-plane bounded by $x = 0$, $y = 1$, and $y = x^2$. with $x \geq 0$

(a) Sketch R.

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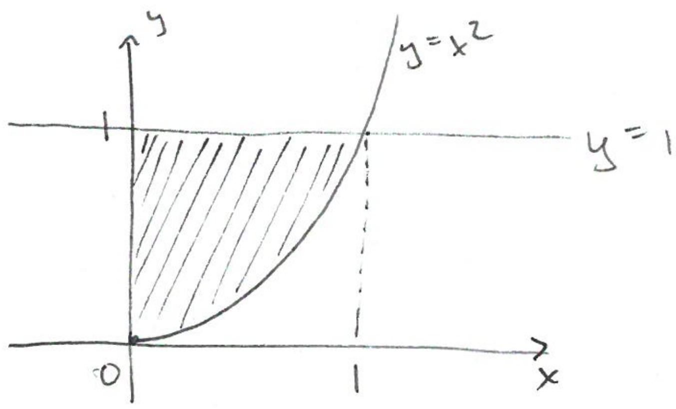

(b) Find the volume of the solid over R bounded above by $f(x, y) = 1 + \frac{x}{(1+y)^2}$.

$$\text{Volume} = \int_0^1 \int_{x^2}^1 \left(1 + \frac{x}{(1+y)^2}\right) dy\, dx$$

$$\int_{x^2}^1 \left(1+\frac{x}{(1+y)^2}\right) dy = \left(y - \frac{x}{1+y}\right)\Big|_{x^2}^1$$

$$= \left(1 - \frac{x}{2}\right) - \left(x^2 - \frac{x}{1+x^2}\right)$$

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$$\text{So Volume} = \int_0^1 \left(1 - \frac{1}{2}x - x^2 + \frac{x}{1+x^2}\right) dx$$

$$= x - \frac{1}{4}x^2 - \frac{1}{3}x^3 + \frac{1}{2}\ln(1+x^2)\Big]_0^1$$

$$= 1 - \frac{1}{4} - \frac{1}{3} + \frac{1}{2}\ln(2) - \left(0 + \frac{1}{2}\ln 1\right)$$

$$= \boxed{\frac{5}{12} + \frac{1}{2}\ln 2}$$

v6.1