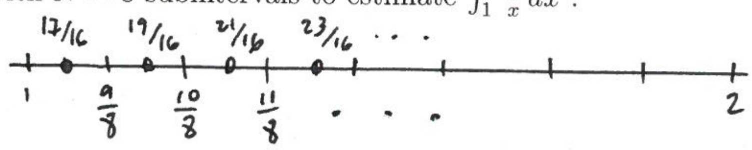
Math 1251: Worksheet #7 **Winter 2018**

Name: SOLUTIONS	A#:	Section:

1. Use your calculator to compute the following to **6 decimal places of accuracy**.

(a) Use the midpoint rule with $N = 8$ subintervals to estimate $\int_1^2 \frac{1}{x}\,dx$.

$$\Delta x = \frac{2-1}{8} = \frac{1}{8}$$



4

$$\int_1^2 \frac{1}{x}\,dx \approx \left(\frac{16}{17}+\frac{16}{19}+\frac{16}{21}+\frac{16}{23}+\frac{16}{25}+\frac{16}{27}+\frac{16}{29}+\frac{16}{31}\right)\cdot\frac{1}{8}$$

$$= 2\left(\frac{1}{17}+\frac{1}{19}+\frac{1}{21}+\cdots+\frac{1}{31}\right)$$

$$\approx \boxed{0.692661}$$

(b) Use the trapezoid rule with $N = 8$ subintervals to estimate $\int_1^2 \frac{1}{x}\,dx$.

$$\int_1^2 \frac{1}{x}\,dx \approx \left(1+2\cdot\frac{8}{9}+2\cdot\frac{8}{10}+2\cdot\frac{8}{11}+2\cdot\frac{8}{12}+2\cdot\frac{8}{13}+2\cdot\frac{8}{14}+2\cdot\frac{8}{15}+\frac{1}{2}\right)\cdot\frac{1/8}{2}$$

4

$$= \frac{1}{16}\left(\frac{3}{2}+16\left(\frac{1}{9}+\frac{1}{10}+\frac{1}{11}+\cdots+\frac{1}{15}\right)\right)$$

$$\approx \boxed{0.694122}$$

(c) Apply Simpson's rule with $N = 8$ subintervals to estimate $\int_1^2 \frac{1}{x}\,dx$.

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$$\int_1^2 \frac{1}{x}\,dx \approx \frac{2M+T}{3} \approx \frac{2(0.692661)+0.694122}{3}$$

$$\approx 0.693148$$

(d) Find the true value of $\int_1^2 \frac{1}{x}\,dx$.

$$\int_1^2 \frac{1}{x}\,dx = \ln|x|\Big]_1^2 = \ln 2 - \ln 1$$

2

$$= \ln 2$$

$$\approx \boxed{0.693147}$$

2. Estimate the monthly payment on a \$200000 mortgage amortized over 25 years at an interest rate of 4% by viewing it as an outlay of cash at a constant rate under continuous compounding.

Say we pay \$Y per year.

Then we need $200000 = \int_0^{25} Ye^{-0.04t}\,dt$

$$= Y\int_0^{25} e^{-t/25}\,dt$$

$$= -25Ye^{-t/25}\Big]_0^{25}$$

$$= -25Y(e^{-1}-1)$$

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So $Y = \dfrac{200000}{25(1-e^{-1})}$

$$= \frac{8000}{1-e^{-1}} \approx \$12655.81$$

Monthly payment is approximately $\frac{Y}{12} \approx \boxed{\$1054.65}$

3. A large city has a population density of approximately $40000e^{-x/2}$ people per km^2 at a distance of x kilometres from the city centre. Estimate the total population of the city.

Population $\approx \int_0^{\infty} 2\pi x(40000e^{-x/2})\,dx$

$$= 80000\pi \int_0^{\infty} xe^{-x/2}\,dx$$

Bonus +3

Now: $\int xe^{-x/2}\,dx = -2xe^{-x/2} - \int(-2e^{-x/2})\,dx$

$$= -2xe^{-x/2} - 4e^{-x/2} + C$$

(Using integration by parts, $u = x$, $du = dx$, $dv = e^{-x/2}dx$, $v = -2e^{-x/2}$)

So $\int_0^{\infty} xe^{-x/2}\,dx = \lim_{b\to\infty} \int_0^b xe^{-x/2}\,dx$

$$= \lim_{b\to\infty} \left(-2xe^{-x/2} - 4e^{-x/2}\right)\Big|_0^b$$

$$= \lim_{b\to\infty} \left(\cancel{-2be^{-b/2}} - \cancel{4e^{-b/2}} + 4\right) = 4$$

Thus population $\approx 80000\pi(4) \approx \boxed{1005310}$

v7.1