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Math 1251: Worksheet #8 **Winter 2018**

Name: SOLUTIONS	A#:	Section:

1. (a) Evaluate $\int_1^\infty \frac{x^2}{(1+7x^3)^{4/3}}\,dx$.

Let $\begin{cases} u = 1+7x^3 \\ du = 21x^2\,dx \end{cases}$ to get $\int \frac{x^2}{(1+7x^3)^{4/3}}\,dx = \frac{1}{21}\int u^{-4/3}\,du$

$= \frac{1}{21}\left(-3u^{-1/3}\right) + C$

$= -\frac{1}{7}\left(1+7x^3\right)^{-1/3} + C$

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Then $\int_1^\infty \frac{x^2}{(1+7x^3)^{4/3}} = \lim_{b\to\infty} -\frac{1}{7}\left(1+7x^3\right)^{-1/3}\Big|_1^b = \lim_{b\to\infty}\left(\frac{-1}{7(1+7b^3)^{1/3}} + \frac{1}{7\cdot 8^{1/3}}\right)$

$= 0 + \frac{1}{14} = \boxed{\frac{1}{14}}$

(b) Use your answer to (a) to decide whether $\int_1^\infty \frac{x^2}{(1+7x^4)^{4/3}}\,dx$ converges or diverges. **Briefly explain your reasoning.**

It converges, because part (a) shows $\int_1^\infty \frac{x^2}{(1+7x^3)^{4/3}}\,dx$ converges, and $\frac{x^2}{(1+7x^4)^{4/3}} < \frac{x^2}{(1+7x^3)^{4/3}}$.

2. Evaluate $\int_{-\infty}^\infty \frac{e^x}{(e^x+3)^2}\,dx$.

Let $\begin{cases} u = e^x+3 \\ du = e^x\,dx \end{cases}$ to get $\int \frac{e^x}{(e^x+3)^2}\,dx = \int \frac{du}{u^2} = -\frac{1}{u} + C$

$= \frac{-1}{e^x+3} + C$

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Then $\int_{-\infty}^0 \frac{e^x}{(e^x+3)^2}\,dx = \lim_{a\to-\infty} -\frac{1}{e^x+3}\Big]_a^0 = \lim_{a\to-\infty}\left(-\frac{1}{4} + \frac{1}{e^a+3}\right) = -\frac{1}{4}+\frac{1}{3} = \frac{1}{12}$

$\int_0^\infty \frac{e^x}{(e^x+3)^2}\,dx = \lim_{b\to\infty} \frac{-1}{e^x+3}\Big]_0^b = \lim_{b\to\infty}\left(\frac{-1}{e^b+3} + \frac{1}{4}\right) = 0 + \frac{1}{4} = \frac{1}{4}$

So $\int_{-\infty}^\infty \frac{e^x}{(e^x+3)^2}\,dx = \int_{-\infty}^0 \frac{e^x}{(e^x+3)^2}\,dx + \int_0^\infty \frac{e^x}{(e^x+3)^2}\,dx$

$= \frac{1}{12} + \frac{1}{4}$

$= \boxed{\frac{1}{3}}$

3. Below is the slope field for the equation $y' = xy^2+1$. Sketch two solutions of this equation; one satisfying $y(0) = 2$, and the other satisfying $y(0) = -2$.

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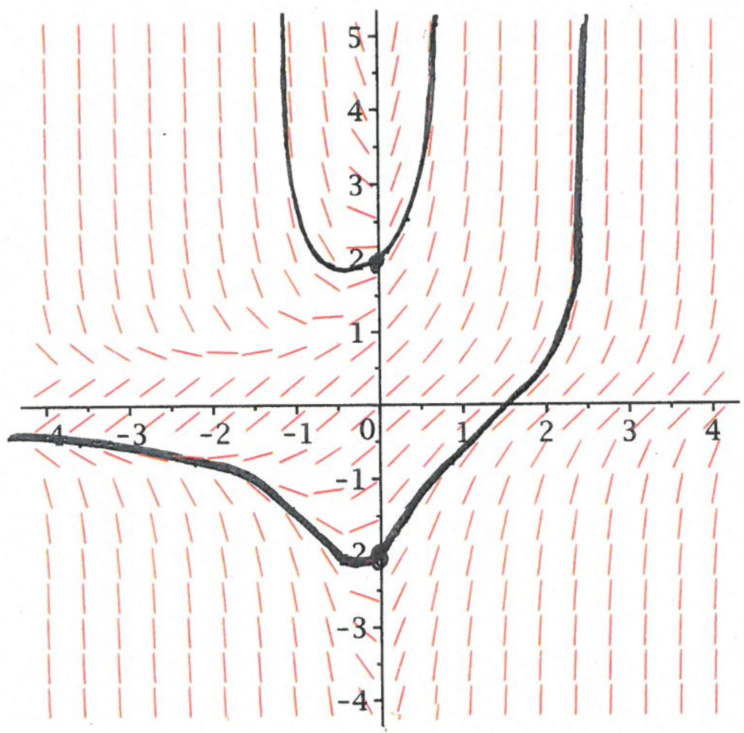

4. Find all solutions $y(t)$ of the differential equation $(t-1)y' = (y-1)t$.

$$(t-1)\frac{dy}{dt} = (y-1)t \Rightarrow \frac{1}{y-1}\,dy = \frac{t}{t-1}\,dt$$

$$\Rightarrow \int \frac{1}{y-1}\,dy = \int \frac{t}{t-1}\,dt$$

Or by substituting $u = t-1$..

$$\Rightarrow \ln(y-1) = \int \frac{(t-1)+1}{t-1}\,dt$$

$$= \int \left(1 + \frac{1}{t-1}\right)dt$$

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$$= t + \ln(t-1) + C_0$$

$$\Rightarrow y-1 = e^{t+\ln(t-1)+C_0}$$

$$= e^{C_0} e^{t} e^{\ln(t-1)}$$

$$\Rightarrow \boxed{y = Ce^t(t-1)+1}, \quad C = e^{C_0}$$

Note: I have ignored a number of technical details here.

- We should have checked $y-1 \neq 0$ before dividing by $y-1$
- We should have $\int \frac{1}{y-1}\,dy = \ln|y-1|$ etc.

If you check all the details, you get the same answer as above, but with C any constant $(+,-,0)$. v8.1

Ask me if you want an explanation.