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Math 1251: Worksheet #9 **Winter 2018**

Name: SOLUTIONS	A#:	Section:

1. Solve the initial-value problem $(t^2+1)y' = t(y+t^2)$, $y(0) = 5$.

Rearrange: $(t^2+1)y' - ty = t^3$

$$\Rightarrow \quad y' - \frac{t}{t^2+1}y = \frac{t^3}{t^2+1}$$

Integrating factor is:

$$A(t) = e^{\int \frac{-t}{t^2+1}dt} = e^{-\frac{1}{2}\ln(t^2+1)}$$
$$= e^{\ln(t^2+1)^{-1/2}}$$
$$= (t^2+1)^{-1/2}$$

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Then $$y(t) = \frac{1}{(t^2+1)^{-1/2}} \int (t^2+1)^{-1/2} \cdot \frac{t^3}{t^2+1}\,dt$$
$$= (t^2+1)^{1/2} \int \frac{t^3}{(t^2+1)^{3/2}}\,dt$$

Let $u = t^2+1$, so $du = 2t\,dt$ and $t^2 = u-1$

to get $$\int \frac{t^3}{(t^2+1)^{3/2}}\,dt = \frac{1}{2}\int \frac{u-1}{u^{3/2}}\,du$$
$$= \frac{1}{2}\int (u^{-1/2} - u^{-3/2})\,du$$
$$= \frac{1}{2}(2u^{1/2} + 2u^{-1/2} + C)$$
$$= (t^2+1)^{1/2} + (t^2+1)^{-1/2} + \frac{C}{2}$$

So: $$y(t) = (t^2+1) + 1 + \frac{C}{2}(t^2+1)^{1/2}$$

Then $y(0) = 5 \Rightarrow 5 = 1 + 1 + \frac{C}{2} \Rightarrow C = 6$

Therefore $$\boxed{y(t) = t^2 + 2 + 3(t^2+1)^{1/2}}$$

v9.1

2. Find $\int x\left(\sqrt[3]{x} - \frac{5}{x^2} + 1\right)\, dx$

$$= \int \left(x^{4/3} - \frac{5}{x} + x\right)$$

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$$= \boxed{\frac{3}{7}x^{7/3} - 5\ln|x| + \frac{1}{2}x^2 + C}$$

3. Find $\int \frac{e^x + x - 1}{e^{3x}}\, dx$

$$= \int \left(e^{-2x} + xe^{-3x} - e^{-3x}\right) dx$$

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$$= -\frac{1}{2}e^{-2x} + \frac{1}{3}e^{-3x} + \int xe^{-3x}\, dx$$

Use integration by parts: $u = x$ $\quad dv = e^{-3x}\,dx$
$du = dx$ $\quad v = -\frac{1}{3}e^{-3x}$

Get $\int xe^{-3x}\,dx = -\frac{1}{3}xe^{-3x} + \frac{1}{3}\int e^{-3x}\,dx$

$$= -\frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x} + C$$

So $\int \frac{e^x + x - 1}{e^{3x}}\,dx = -\frac{1}{2}e^{-2x} + \frac{1}{3}e^{-3x} - \frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x} + C$

$$= \boxed{-\frac{1}{2}e^{-2x} + \frac{2}{9}e^{-3x} - \frac{1}{3}xe^{-3x} + C}$$