**Math 1251: Worksheet #1** **Winter 2018**

| Name: | A#: | Section: |
|---|---|---|

1. $\int \left(2x^5 - \frac{3}{4x^2} + \frac{e}{5}\right) dx$

2. $\int e^x(1 + e^{4x})\, dx$

3. $\int \frac{\sin \pi x}{2}\, dx$

4. $\int \frac{4 - x}{\sqrt[3]{x}}\, dx$

5. Find all $k$ such that $\int \frac{dx}{1-x^2} = k \ln\left(\frac{x+1}{x-1}\right) + C$ [*Hint:* First use logarithm rules!]

6. The function $y(x)$ satisfies $\frac{d^2y}{dx^2} = 5x^{2/3} - 1$ and $y(0) = -1$, $y(1) = 0$. Find $y(x)$.

**Math 1251: Worksheet #2** **Winter 2018**

| Name: | A#: | Section: |
|---|---|---|

1. $\displaystyle\int_{-1}^{1} \frac{3}{e^{2t}}\, dt$

2. $\displaystyle\int_{0}^{1} (1 - x^2)^2\, dx$

3. Given that $\displaystyle\int_{-1}^{2} f(x)\, dx = 3$ and $\displaystyle\int_{-1}^{6} f(x)\, dx = 5$, find $\displaystyle\int_{2}^{6} f(x)\, dx$.

4. Find all real numbers $b > 0$ such that the area under the graph of $y = \dfrac{x^2 + 1}{2}$ over the interval $0 \le x \le b$ is equal to $b$..

5. (a) Approximate the area under the graph of $y = 1/x^2$ between $x = 1$ and $x = 3$ by a Riemann sum, with $N = 6$ subintervals and using left endpoints. You can leave your answer as a sum of fractions.

(b) Illustrate your approximation in (a) with a sketch. Use the sketch to determine whether you have over- or underestimated the area.

(c) Find the true value of the area described in (a).

6. A water tank is being drained. After $t$ minutes of draining, the water level in the tank is decreasing at a rate of $(8 - t)^{-2/3}$ metres per minute. How much does the water level decrease in the first 7 minutes?

**Math 1251: Worksheet #3** **Winter 2018**

| **Name:** | **A#:** | **Section:** |
|---|---|---|

1. Find the volume of the solid obtained by revolving the segment of the curve $y = x + \frac{1}{x}$ between $x = 1$ and $x = 2$ about the $x$-axis.

2. A particle moves in a straight line so that its velocity after $t$ seconds is $v(t) = t^3 - 3t^2 + 2t$ metres per second.

   (a) Find the net displacement of the particle over the time interval $1 \leq t \leq 3$.

   (b) Find the total distance travelled by the particle over the time interval $1 \leq t \leq 3$.

3. Sketch the curves $y = 4 - x$ and $y = -x^2 + 2x + 8$ and find the area bounded between them. (Show all your work.)

4. Find all $k$ such that the average value of $f(x) = ke^{x/2} - 1$ over $[0, 2]$ is equal to 2.

**Math 1251: Worksheet #4** **Winter 2018**

| Name: | A#: | Section: |
|---|---|---|

1. Let $f(x, y, z) = x^3 e^{z+xy^2} + (x - z)^3$.

   (a) Compute $\dfrac{\partial f}{\partial x}$

   (b) Compute $\dfrac{\partial f}{\partial y}$

   (c) Compute $\dfrac{\partial f}{\partial z}$

   (d) Verify that $\dfrac{\partial^2 f}{\partial y\, \partial x} = \dfrac{\partial^2 f}{\partial x\, \partial y}$.

2. Let $f(x, y) = x^2(1 - xy)$. On separate pairs of axes, sketch the cross-sections $z = f(x, 1)$ and $z = f(1, y)$.

3. Let $f(x, y) = \dfrac{y - 1}{x^2 + 1}$. Draw the level curves $f(x, y) = c$ for $c = 0, 1, 2$.

4. Let $g(x, y) = (1 - x^3\sqrt{y})^4$. Use partial derivatives to estimate the value of $g(1 + h, 4)$ for small $h$.

**Math 1251: Worksheet #5** **Winter 2018**

| **Name:** | **A#:** | **Section:** |
|---|---|---|

1. Let $f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 1$.

   (a) Find all critical points of $f$ (i.e. points where $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0$).

(b) Use the second-derivative test to classify the critical points of $f$ as local maxima, local minima, or neither.

2. Find the least-squares line for the data points $(-1, -5), (1, 0), (3, 4), (5, 7)$.

**Math 1251: Worksheet #6** **Winter 2018**

| **Name:** | **A#:** | **Section:** |
|---|---|---|

1. Compute the following integrals:

   (a) $\displaystyle\int \frac{dx}{x \ln x}$

   (b) $\displaystyle\int \frac{x^2}{(1-2x^3)^4}\,dx$

   (c) $\displaystyle\int \frac{dt}{\sqrt{t}e^{\sqrt{t}}}$

(d) $\int_0^1 x\sqrt{1+3x}\,dx$

2. Let $R$ be the region of the $(x, y)$-plane bounded by $x = 0$, $y = 1$, and $y = x^2$.

(a) Sketch $R$.

(b) Find the volume of the solid over $R$ bounded above by $f(x, y) = 1 + \dfrac{x}{(1+y)^2}$.

**Math 1251: Worksheet #7** **Winter 2018**

| **Name:** | **A#:** | **Section:** |
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1. Use your calculator to compute the following to **6 decimal places of accuracy**.

   (a) Use the midpoint rule with $N = 8$ subintervals to estimate $\int_1^2 \frac{1}{x}\,dx$ .

   (b) Use the trapezoid rule with $N = 8$ subintervals to estimate $\int_1^2 \frac{1}{x}\,dx$ .

   (c) Apply Simpson's rule with $N = 8$ subintervals to estimate $\int_1^2 \frac{1}{x}\,dx$ .

   (d) Find the true value of $\int_1^2 \frac{1}{x}\,dx$.

2. Estimate the monthly payment on a \$200000 mortgage amortized over 25 years at an interest rate of 4% by viewing it as an outlay of cash at a constant rate under continuous compounding.

3. A large city has a population density of approximately $40000e^{-x/2}$ people per $\text{km}^2$ at a distance of $x$ kilometres from the city centre. Estimate the total population of the city.

**Math 1251: Worksheet #8** **Winter 2018**

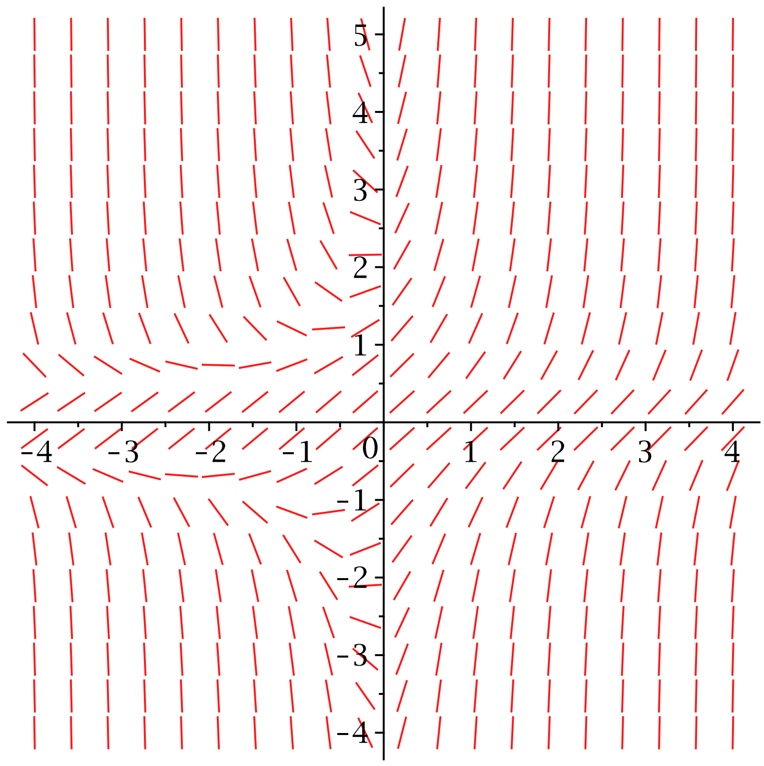
| **Name:** | **A#:** | **Section:** |
|---|---|---|

1. (a) Evaluate $\displaystyle\int_1^\infty \frac{x^2}{(1+7x^3)^{4/3}}\,dx$.

   (b) Use your answer to (a) to decide whether $\displaystyle\int_1^\infty \frac{x^2}{(1+7x^4)^{4/3}}\,dx$ converges or diverges. **Briefly explain your reasoning.**

2. Evaluate $\displaystyle\int_{-\infty}^\infty \frac{e^x}{(e^x+3)^2}\,dx$.

3. Below is the slope field for the equation $y' = xy^2 + 1$. Sketch two solutions of this equation; one satisfying $y(0) = 2$, and the other satisfying $y(0) = -2$.




4. Find all solutions $y(t)$ of the differential equation $(t-1)y' = (y-1)t$.

v8.1

| **Name:** | **A#:** | **Section:** |
|---|---|---|

1. Solve the initial-value problem $(t^2+1)y' = t(y+t^2), \quad y(0) = 5.$

2. Find $\int x\left(\sqrt[3]{x} - \frac{5}{x^2} + 1\right)\, dx$

3. Find $\int \frac{e^x + x - 1}{e^{3x}}\, dx$