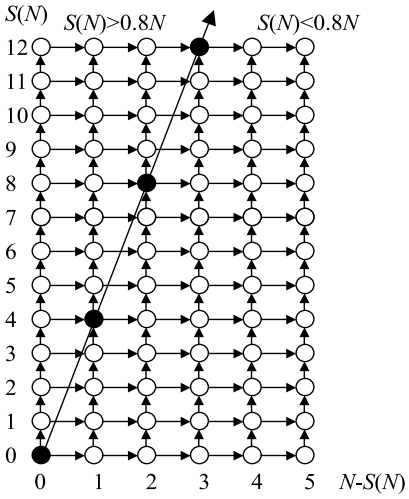
Putnam Proof Without Words

Robert J. MacG. Dawson[1]
Saint Mary's University
Halifax, Nova Scotia,
Canada B3H 3C3
rdawson@stmarys.ca

From the 2004 William Lowell Putnam Mathematical Competition, A1:

> Basketball star Shanille O'Keal's team statistician keeps track of the number, $S(N)$, of successful free throws she has made in her first N attempts of the season. Early in the season, $S(N)$ was less than 80% of N, but by the end of the season, $S(N)$ was more than 80%. Was there necessarily a moment in between when $S(N)$ was exactly 80% of N?
>
> Answer: Yes. Proof:



> The reader should now also be able to answer the following "riders" that did not form part of the competition question:
>
> (a) Answer the same question assuming that Shanille had $S(N) > 0.8N$ early in the season and $S(N) < 0.8N$ at the end.
> (b) What other values could be substituted for 80% in the original question?

[1]Supported by NSERC Canada