Nova Scotia Math League

2010–2011

Game One

PROBLEMS

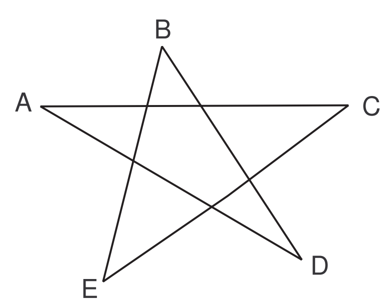
Team Questions

1) Find the sum of all positive divisors of 108.

   (For example, the number 6 has four positive divisors, namely 1, 2, 3, and 6, and their sum is 12.)

2) The numbers $x,\ 2x+4$, and $3x+6$, in that order, form three consecutive terms of a geometric sequence. Find the next term of the sequence.

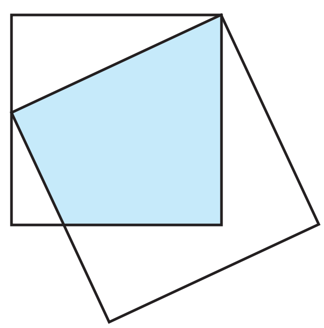
3) In the diagram below, the acute angle at A is 20° and the acute angle at D is 30°. Determine the *sum* of the acute angles at A, B, C, D, and E.



4) For certain real numbers a and b it is known that $x = 1 - 2i$ is one of the roots of the equation $3x^2 + ax + b = 0$ (where, as usual, $i = \sqrt{-1}$). Determine a and b.

5) An election is held between two candidates, A and B. When results are tallied, it is observed that the first 50 ballots are split evenly between the candidates, but thereafter only one in five ballots is in favour of candidate B. Given that A receives exactly three times as many votes as B overall, determine the total number of votes cast.

6) Two squares overlap as shown in the figure. Given that squares have areas 25 and 29, determine the area of the shaded region.



7) You have three standard six-sided dice, two of them blue and one red. Upon throwing the dice, what is the probability that the number showing on the red die is at least as great as the sum of those on the blue dice?

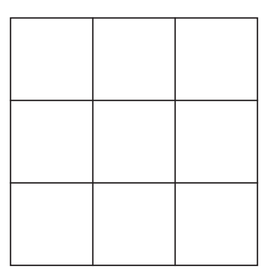
8) Determine the sum of the coefficients in the expansion of $(x - 2y + 3z)^4$.

(For example, $(2x - 3y)^2$ expands to give $4x^2 - 12xy + 9y^2$, and the sum of the coefficients is $4 + (-12) + 9 = 1$.)

9) A 4 metre length of wire is cut into two pieces. One piece is bent to form a square, and the other is bent to form a rectangle that is three times longer than it is wide. What is the minimum possible area enclosed by these two figures?

(That is, you are to determine the smallest possible sum of the areas of the square and rectangle.)

10) The numbers 1 through 9 are to be entered into the squares of the grid below (each number used exactly once, one number per square). In how many ways can this be done so that 1 and 9 do not appear in the same row or column?



Pairs Relay

A. Each interior angle of a regular polygon is 144°. Let $\mathtt{A}$ be the number of sides of the polygon.

Pass on $\mathtt{A}$

B. A city reports the following year-over-year changes in population for four consecutive years: an increase of 25%, an increase of 40%, a decrease of 20%, and a decrease of $\mathtt{A}$%.

Let $\mathtt{B}$ be the net percent increase in population over the entire four year span.

Pass on $\mathtt{B}$

C. You will receive $\mathtt{B}$.

Let $\mathtt{C}$ be the units digit of $1! + 2! + 3! + \cdots + \mathtt{B}!$.

Pass on $\mathtt{C}$

D. You will receive $\mathtt{C}$.

The points $P = (1, \mathtt{C})$, $Q = (-1, -2)$, $R = (5, 1)$, and $S = (x, y)$ are the vertices of a parallelogram in the plane, with P and R being diagonally opposite. Let $\mathtt{D} = x + y$.

Done!

Individual Relay

A. Let A be the smallest positive integer such that 54A is a perfect square.

Pass on A

B. You will receive A.

The average of a set of A numbers is 17. When one number is removed from the set, the average changes to 18. Let B be the number that was removed.

Pass on B

C. You will receive B.

Fifty points are equally spaced around the circumference of a circle and are labeled with the numbers 1 through 50, in order around the circle. Let C be the number diametrically opposite to B.

Pass on C

D. You will receive C.

Let D be the number of pairs of positive integers (x, y) that satisfy $2x + 3y = \texttt{C}$.

Done!