Nova Scotia Math League

2012–2013

Game One

PROBLEMS

Team Questions

1. Angela, Bev, and Clara compete in a 10km race. Each girl runs at a constant speed throughout the race. Angela is the winner, beating Bev by 2km and beating Clara by 4km.

   By how many kilometres does Bev beat Clara?

2. Woody wins a logging contract to remove oak trees (and *only* oak trees) from a particular forest. Initially, 99% of the trees in the forest are oak. When Woody completes his contract, 98% of the remaining trees are oak.

   What percentage of the trees did Woody remove from the forest?

3. How many distinct ways are there to fill the squares of a 3×3 grid with the numbers 1, 2, and 3, so that no digit appears more than once in the same row or the same column?

4. There are 4 red balls and 3 green balls placed at random in a line. Find the probability that both balls at the ends are green.

5. How many distinct points in the (x, y)-plane are equidistant from the x-axis, the y-axis, and the line $x + y = 2013$?

6. The function f is defined on the positive integers as follows:

$$f(n) = \begin{cases} 3n+1 & \text{if } n \text{ is odd,} \\ n/2 & \text{if } n \text{ is even.} \end{cases}$$

For example, $f(9) = 28$ and $f(10) = 5$.

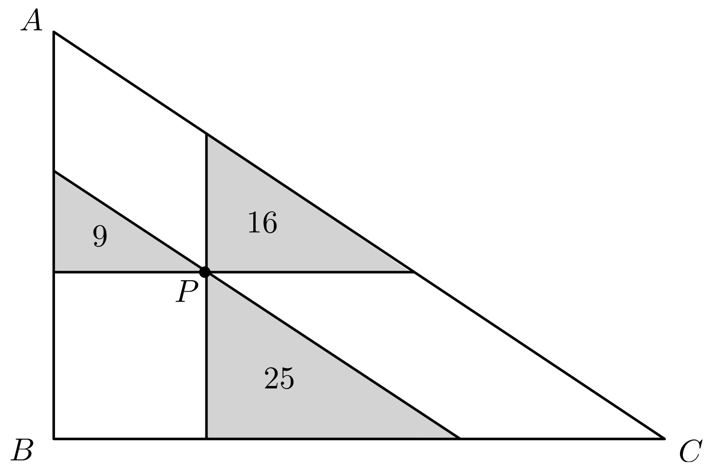
Find the unique **odd** positive integer k such that $f(f(f(k))) = 31$.

7. Evaluate the following product:

$$(\sqrt{2}+\sqrt{3}+\sqrt{5})(-\sqrt{2}+\sqrt{3}+\sqrt{5})(\sqrt{2}-\sqrt{3}+\sqrt{5})(\sqrt{2}+\sqrt{3}-\sqrt{5})$$

8. In a parallelogram, the sides have lengths 11 and 13, and one diagonal has length 18. Find the length of the other diagonal.

9. In the figure below, triangle ABC has $\angle B = 90^\circ$ and $|AB| : |BC| = 3 : 4$. Point P lies in the interior of $\triangle ABC$, and three lines have been drawn through P parallel with the sides of the triangle.



If the small shaded triangles have areas 9, 16, and 25, as indicated, what is the area of $\triangle ABC$?

10. The line $y = x + 1$ intersects the ellipse

$$\frac{x^2}{2012} + \frac{y^2}{2013} = 1$$

in two points, A and B. Find the (x, y) coordinates of the midpoint of line segment AB.

Pairs Relay

P-A. For a positive integer n, let $s(n)$ and $p(n)$ denote the **sum** and **product** of the digits of n, respectively. For example, $s(35) = 3 + 5 = 8$ and $p(35) = 3 \cdot 5 = 15$.

The two-digit integer M has the property that $s(M) + p(M) = M$.

Let $\mathtt{A}$ be the units digit of M. **Pass on A**

P-B. You will receive $\mathtt{A}$.

Alice and Bob each independently choose a random integer between 1 and $\mathtt{A}$, inclusive. Let $\mathtt{B}$ be the probability that Alice and Bob do not select the same number.

Pass on B

P-C. You will receive $\mathtt{B}$.

Suppose x and y are real numbers such that $x + y = 2$ and $xy = \mathtt{B}$. Let

$$\mathtt{C} = \frac{x}{y} + \frac{y}{x}.$$

Pass on C

P-D. You will receive $\mathtt{C}$.

The following sequence consists of all positive integers that are **not** perfect squares or perfect cubes:

$$2, 3, 5, 6, 7, 10, 11, 12, 13, \ldots$$

Let $n = 50\mathtt{C}$, and let $\mathtt{D}$ be the n-th term of this sequence.

Done!

Individual Relay

I-A. Let $\mathtt{A}$ be the number of ways the letters of the word BOAT can be arranged so that the vowels are not side-by-side.

Pass on $\mathtt{A}$

I-B. You will receive $\mathtt{A}$.

Let $\mathtt{B}$ be the unique positive number such that

$$\frac{\mathtt{A}}{\mathtt{B}} - \frac{\mathtt{B}}{\mathtt{A}} = \frac{\mathtt{A}+\mathtt{B}}{\mathtt{A}}.$$

Pass on $\mathtt{B}$

I-C. You will receive $\mathtt{B}$.

Let $\mathtt{C}$ be the area of the region bounded between the lines $y = -3x$, $y = 6x$, and $y = \mathtt{B}$.

Pass on $\mathtt{C}$

I-D. You will receive $\mathtt{C}$.

Suppose x, y and z are nonnegative integers such that $x + y + z = \mathtt{C}$.

Let $\mathtt{D}$ be the maximum possible value of $xyz + xy + yz + zx$.

Done!