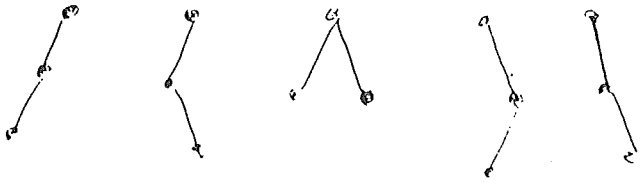
Binary Trees with 1 Node

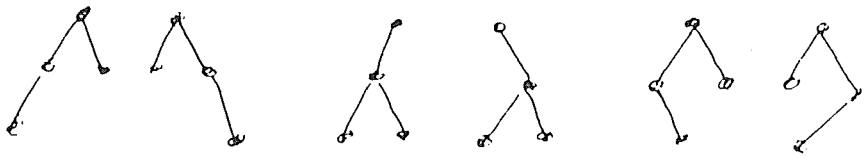
Binary Trees with 2 Nodes

Binary Trees with 3 Nodes

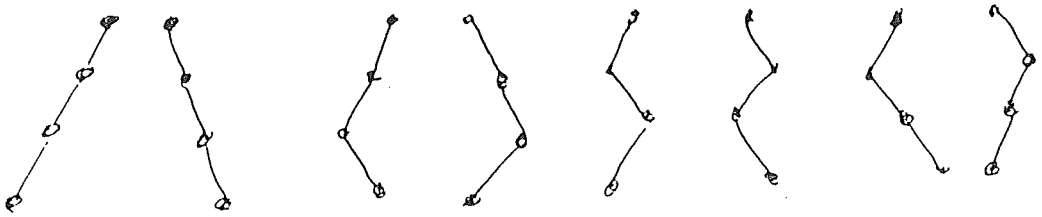


Binary Trees with 4 Nodes

3 levels (6)



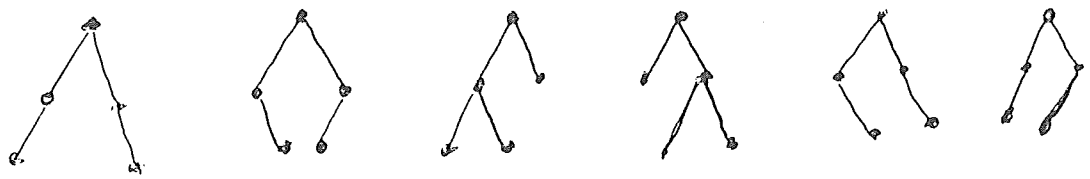
4 levels (5)



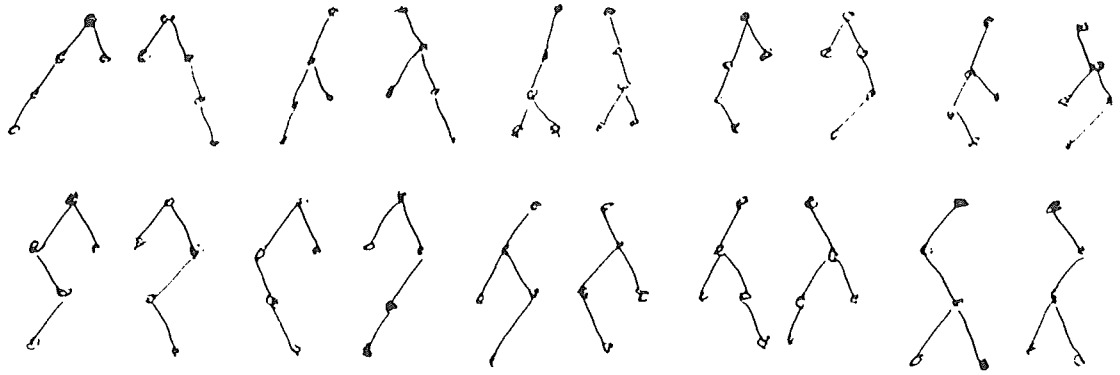
Total = 6 + 5 = 14

Binary Trees with 5 Nodes

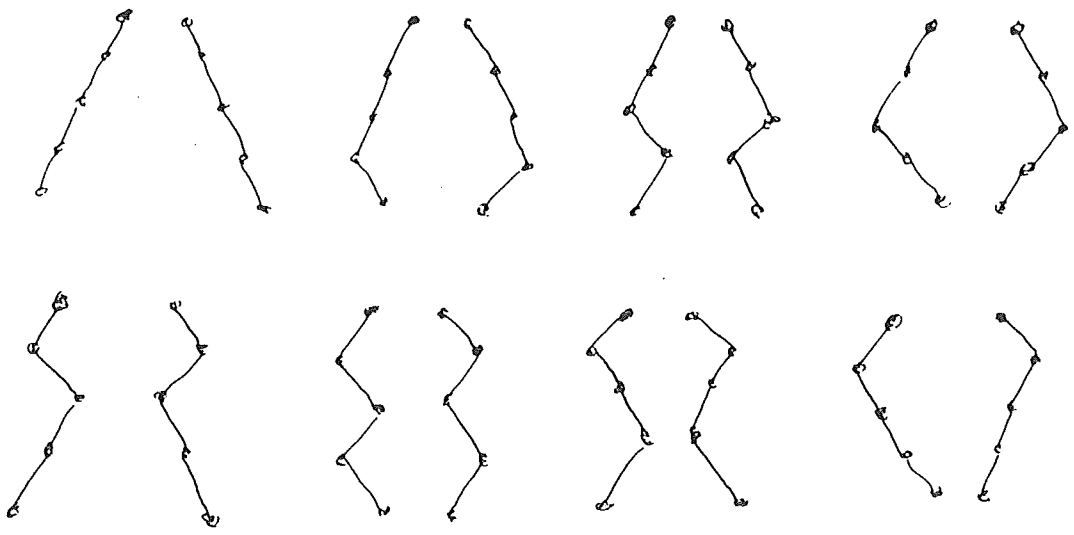
3 levels (6)



4 levels (20)



5 levels (16)



Total = $6 + 20 + 16 = 42$

General Formula: The number of binary trees with n nodes is $\frac{(2n)!}{(n+1)!\,n!}$. (Catalan Numbers)

More questions to which the answer is "Catalan Numbers" (Cat(n))

1. How many "well-formed" sequences of n left and n right parentheses are there?

   Example: $n=3$ Remember: $Cat(3)=5$

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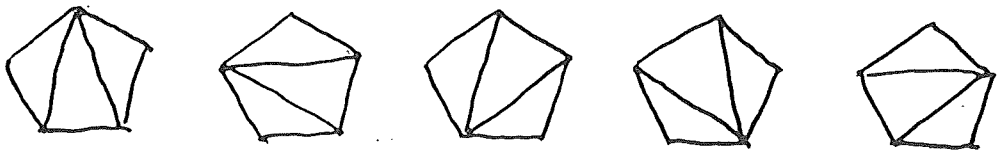
   (( ))( ) ( )(( ))

2. How many ways can we divide a convex polygon with $n+2$ sides into triangles by drawing $n-1$ non-intersecting diagonals?

   Example: $n=3$ (again) Remember: $Cat(3)=5$ (again)



Exercise(s):

Try each of the above for $n=4$.